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ABSTRACT

This paper argues that a major aspect of intelligence is the ability to solve problems and that careful analysis of problem-solving behavior is a means of specifying many of the psychological processes that make up intelligence. The focus is on the mechanisms involved when, in the absence of complete instruction, a person must "invent" a new solution to a problem by assembling previously learned skills. To describe this type of problem-solving behavior the authors set forth an information processing model characterized by three classes of processes: problem detection, feature scanning, and goal analysis. A series of studies on invention, in which children are taught component skills and their behavior is examined in situations where these skills must be combined in a novel fashion, elucidates this model. (Author)

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PROBLEM SOLVING AND INTELLIGENCE

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Abstract

This paper argues that a major aspect of intelligence is the ability to solve problems and that careful analysis of problem solving behavior is a means of specifying many of the psychological processes that make up intelligence. The focus is on the mechanisms involved when, in the absence of complete instruction, a person must "invent" a new solution to a problem by assembling previously learned skills. To describe this type of problem solving behavior the authors set forth an information processing model characterized by three classes of processes: problem detection, feature scanning, and goal analysis. A series of studies on invention, in which children are taught component skills and their behavior is examined in situations where these skills must be combined in a novel fashion, elucidate this model.

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PROBLEM SOLVING AND INTELLIGENCE

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We will argue in this paper that a major aspect of intelligence is the ability to solve problems, and that careful analysis of problem solving behavior constitutes a means of specifying many of the psychological processes that intelligence comprises. To build the argument for this approach, we will first consider some general issues surrounding the term intelligence, and then suggest why problem solving provides fertile ground for the experimental study of intelligence.

We begin by provisionally accepting the layman's quite general definition of intelligence as "the ability to learn," and assume that this means the ability to learn things important in one's environment. We next note that much, perhaps most, learning occurs without formal instruction, that is, out of the context of established teaching institutions. Virtually all of the learning that children manage prior to the age of five or six occurs without such formal instruction. Even during the school years, much of what is learned is outside of any formal curriculum, and during most of one's adult life little formal instruction is engaged in, yet learning certainly continues. Even where deliberate instruction is provided, it is rarely "complete" in terms of assuring that the learner experiences or attends to every aspect of what is to be learned or that he or she is systematically taught every skill in exactly the form used by experts.

This incompleteness of instruction, even in school, is an important point with respect to the definition of intelligence. In traditional forms of schooling (including those most prevalent today), children are exposed about equally to instruction, but some children learn more of whatever is

offered than others. It is precisely this difference in amount learned under approximately equivalent conditions of exposure that makes intelligence tests work as predictive instruments. The actual items on most intelligence tests are tests of what one already knows. Whoever knows more (of the kind of thing being tested) and knows how to use that information under test-like conditions will do better on the tests. Thus, the tests measure what the individual has managed to learn from past exposure, relative to other individuals. Presumably, if someone has been good in the past at acquiring knowledge, and if conditions of instruction remain more or less the same, then he or she will be good at acquiring knowledge in the future. In other words, although the test items are usually measures of learning already achieved, intelligence tests are indirectly measures of how well one can learn on one's own.

The import of these observations is that if we are to account for intelligence, we must account for the ability to learn on one's own in the absence of direct or complete instruction. To put it another way, we must account for the processes involved when an individual, under conditions of limited or less than explicit instruction, makes a transition from one state of competence to another. More broadly, we must seek a characterization of processes that are involved, or that an individual employs, in the acquisition of a new capability.

Attempts to Account for Transitions in Intellectual Competence

Where can we turn in modern psychology for serious study and elucidation of this problem of cognitive transition? An obvious first candidate is cognitive-developmental theory of the Piagetian and neo-Piagetian variety. Cognitive-developmental theory has been centrally concerned with charting changes in intellectual ability, and further, has stressed the "working from within" nature of cognitive growth. The child "constructs" cognitive reality

00006

and these constructions define his intellectual competence. In accounting for transitions in competence, emphasis is placed on what the child does, not on what he is taught.

Although the concern for internally produced structural transition is indeed strong, actual research on cognitive development in the Piagetian tradition has focused largely on describing the resulting state differences--differences in cognitive performances and inferred "deep" competence--at different points in development. This work has left the problem of accounting for transition between these states, or "stages," almost completely uninvestigated and unspecified. The problem is considered only in the most general terms. When a child is in a given stage, he or she assimilates new information and experience into existing cognitive structures. From time to time however, "accommodations" occur, like miniature scientific revolutions. New structures are formed to account for the new experience and these then become the organizing structures for future encounters with the environment. Transition, then, occurs via accommodation. But on closer look, all of the important questions remain. When and why do accommodations take place? What tips the balance in favor of accommodation over assimilation? What actually happens during the process of accommodation? These questions are posed by cognitive-developmental theory, but not answered by it.

A recent monograph by Flavell (1972) underscores the absence of a strong theory of transitions within the Piagetian stage-theory tradition. Flavell is concerned with how two "cognitive items" that are sequentially ordered in terms of temporal development are related to one another, that is, how acquisition of one item influences or determines acquisition of another. Flavell suggests that of several kinds of transition relationships, "inclusion" sequences--cases where items are combined to form new items--are particularly amenable to clear explanation in terms of processes called upon in performance and transition.

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Inclusion relationships have also been proposed as a means of accounting for growth in intellectual capability by Gagné (1962; 1968), in what has come to be known as cumulative learning theory. Gagné argues that complex abilities can be analyzed into simpler components, prerequisites, that are combined during acquisition of the complex ability. Since each prerequisite task can also be analyzed into its component abilities, and since each complex task can be combined with others to produce a still higher level of performance, it is possible to specify a hierarchy of tasks that cumulate through successive layers of positive transfer to greater and greater levels of cognitive competence. With respect to cognitive development, cumulative learning theory suggests that small changes in ability cumulate across tasks and over time to create an apparently large and qualitative shift in competence.

Although at one level of analysis cumulative learning theory seems to function as a potential explanation of stage changes in development, at another and deeper level a learning hierarchy represents no more than a collection of ordered but discrete state descriptions, albeit at a finer grain of description than Piagetian stages. A hierarchy of tasks does not explain the combinatory processes or transfer mechanisms by which new competence is actually produced. It is probably not too extreme to argue that the most interesting events, in terms of a theory of intelligence, happen between the specified points in a hierarchy. Yet cumulative learning theory, like Piagetian theory, is largely silent as to what goes on.

David Klahr (in press) has reported on work that can be thought of as an attempt to formalize through simulation an essentially cumulative learning hypothesis concerning cognitive development. The aim is to characterize increasingly complex performances on Piagetian tasks in terms of a limited set of processes defined by computer programs. These processes, in various combinations, are shown to be sufficient to perform a variety of tasks. As more processes enter the system, more complex and developmentally

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more advanced tasks can be performed. As Klahr points out, current formulations offer no really satisfactory (or "well-modeled") explanations for how a process enters the repertoire. Further, there is, at present, no way to answer the question of how available processes are assembled (i.e., combined) to solve new problems.

This important question of assembly is partially masked by the use of production systems in current modeling attempts.[1] Production systems appear to solve the assembly problem by embedding it in the matching of current short-term memory contents (data structures) with production rule conditions. An action is carried out whenever the proper conditions are met; thus, within a production system formulation, there is no need to postulate a separate assembly mechanism. However, as increasingly complex levels of performance are reached, new production rules are required if actions previously carried out under one set of conditions are to be used under new conditions. Thus, the essential problem of assembly--the use of available actions in new contexts--remains. This point should become clearer as we move to some specific examples later in this paper.

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[1] A production system, as defined by Newell (1973) is a scheme for specifying an information processing system:

> It consists of a set of productions, each production consisting of a condition and an action. It has also a collection of data structures: expressions that encode the information upon which the production system works--on which the actions operate and on which the conditions can be determined to be true or false. A production system, starting with an initially given set of data structures, operates as follows. That production whose condition is true of the current data (assume there is only one) is executed, that is, the action is taken. The result is to modify the current data structures. This leads in the next instant to another (possibly the same) production being executed, leading to still further modification. So it goes, action after action being taken to carry out an entire program of processing, each evoked by its condition becoming true of the momentarily current collection of data structures. The entire process halts either when no condition is true (hence nothing is evoked) or when an action containing a stop operation occurs. (p. 463)

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Assembly as a Mechanism of Transition in Cognitive Competence

The problem before us is accounting for intelligence as a process wherein individuals develop new cognitive competence without direct or complete instruction. The question is one of transition between states of competence. We have argued that neither cognitive-developmental nor cumulative learning theory has as yet adequately addressed the problem of transition, nor have beginning attempts at an information processing theory of development. Further, we have suggested that a useful way of thinking about the problem of transition is to attempt to account for the combination or assembly of existing processes into more complex ones.

Assembly as Problem Solving

If one scans the psychological literature for places where the question of assembly has actually been experimentally addressed, one is drawn to the literature on "problem solving." We are drawn there with some reluctance because problem solving is at least as disorganized a topic in experimental psychology as intelligence and it has not even had its share of psychometricians to provide it with a working operational definition. Problem solving has been studied in one form or another virtually throughout the history of scientific psychology, and proponents of various theories of psychology have attempted to explain problem solving phenomena in terms of their own theoretical constructs. Despite the theoretical diversity, there exists a surprising consensus concerning what constitutes a "problem" in psychological terms, and a review of some of the classical literature on problem solving suggests a number of working hypotheses relevant to our present question of assembly and cognitive transition.

Psychologists agree that the term problem refers to a situation in which an individual is called upon to perform a task not previously encountered and for which externally provided instructions do not specify completely the mode of solution. The particular task, in other words, is new for the

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individual, although processes or knowledge already available can be called upon for solution. Associationist psychologists working in the Hullian tradition, such as Maltzman (1955; Maltzman, Brooks, Bogartz, & Summers, 1958), interpret problem solving in terms of the position of the appropriate response in a habit-family hierarchy. The emphasis is on accessing responses already available, but not dominant. Other definitions, too, stress calling up of available responses, but focus on processes of assembling them to form a new solution. Maier (1933), for example, gives this definition: "The solution of a problem . . . is a pattern consisting of parts of past experience which have become integrated. These parts of experience need never have been previously associated" (p. 144).

Wertheimer (1945/1959), the spokesman for the Gestalt psychology of insight, defines problem solving in the following way:

> [A] discovery does not merely mean that a result is reached which was not known before . . . but rather that a situation is grasped in a new and deeper fashion. . . . These changes of the situation as a whole imply changes in the structural meaning of part items, changes in their place, role and function, which often lead to important consequences. (pp. 169-170)

Wertheimer thus stresses the prior existence of components of the "solution," but focuses on the processes of restructuring and insight that lead to recognition of the solution as relevant.

Problem Solving as Invention

The classical literature on problem solving has directed much of its attention to tasks that require the invention or construction of a new strategy or material object. In these tasks a tool, physical or intellectual, is produced. Materials or processes are combined to make available something that had not existed before. The behavioral and/or technological repertoire is enlarged through processes of cognitive and physical assembly of prior elements.

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The general characteristics of these "invention" tasks can best be conveyed by considering some examples. By far the largest set of such tasks has been studied under the label of "functional fixedness." The most familiar of these include the two-string, hat-rack, pendulum, and blowing-out-the-candle problems introduced by Maier (1970) and the gimlet and candle-on-the-wall problems originally studied by Duncker (1945). In each of these problems, the subject is asked to build an object or to perform some action. An array of objects is provided, one or more of which can be used in solving the problem, but none of which is typically used that way. Thus, there are clamps and poles for the hat-rack problem but no wall hooks. There are various items that can serve as pendulum bobs in the two-string problem, but, at least in some versions of the problem, no extra piece of string or elastic. There are a box and matches in the candle-on-the-wall problem, but the box is filled with tacks and there is no recognizable candleholder.

In all of these tasks objects are combined or assembled in many ways to produce new and (at least temporarily) useful objects. They are inventions in the same sense that the telephone, the Bessemer furnace, and the airplane are inventions. Other invention problems are more cognitive in nature, with the problem solver not necessarily engaging in physical manipulation, but with the same combinatorial processes at work, based on past experience and knowledge as well as current task demands. One such problem is the radiation problem studied and discussed at some length by Duncker (1945). Another is the parallelogram problem studied by Wertheimer (1945/1959) and his students (Luchins & Luchins, 1970).

Invention problems address in a particularly direct way the question of transition in competence without direct instruction--the question with which this paper opened. In invention problems, individuals who are successful solvers have gained a new competence. They can do or make something they were unable to do or make before. They have learned something

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new. Further, they have managed this on their own, or with minimal external help. Thus, they have engaged in learning in the absence of instruction. Finally, in all of the problems the solutions are built out of information or partial solution routines already in the individuals' repertoires. In this way invention problems highlight the assembly process which we have suggested may be central to an understanding of the nature of intelligence.

A Model for Solving Invention Problems

Using commonly current information processing constructs, it is possible to characterize invention/problem solving as a process of encoding a problem, that is, building a representation in working memory (WM), and then searching long-term memory (LTM) for a stored routine (whole or partial) relevant to the problem as formulated. If a routine that works under present conditions of the task environment (TE) is not found, further features of the TE may be noted or the immediate goal of problem solving activity redefined so that routines not previously recognized as relevant or usable will become so. We describe this general set of processes in terms of three aspects: (1) problem detection, (2) feature scanning, and (3) goal analysis.

1. Problem detection. Consider Figure 1. Actions A through F define a problem detection routine. The process assumes an individual who has already encoded the problem as verbally stated. This has established a goal in WM (Box A). The first step in solving the problem consists of searching LTM for a routine that is encoded as relevant to the goal (Box B). If such an item is found, a test is made for whether the conditions required for carrying out the routine are present (C). If the answer is yes, the routine can be performed (D), and then tested for success in meeting the goal (E). If successful, the problem is "solved"--in fact, it was not really a problem since it had usable routines already available. The right-hand branches from B, C, and E, by contrast, set a "true" problem. If at B no

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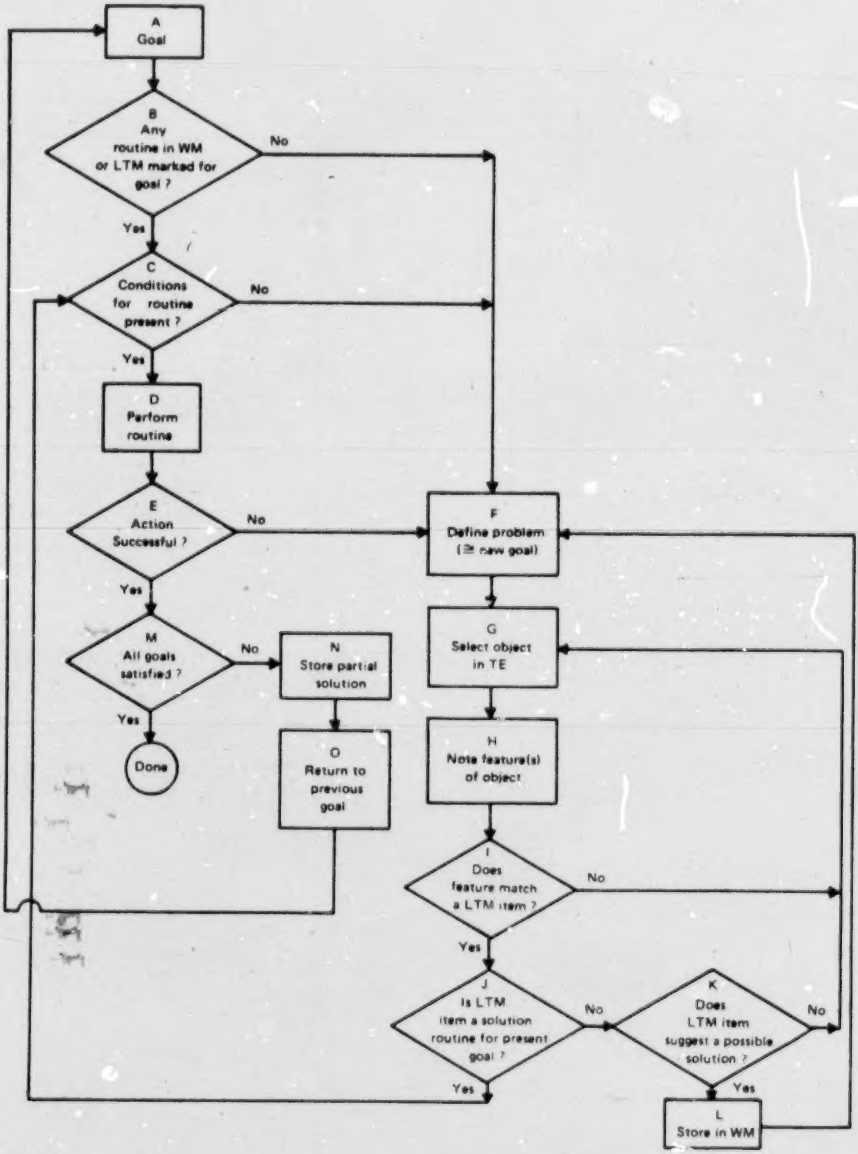

Figure 1. A model for solving invention problems.

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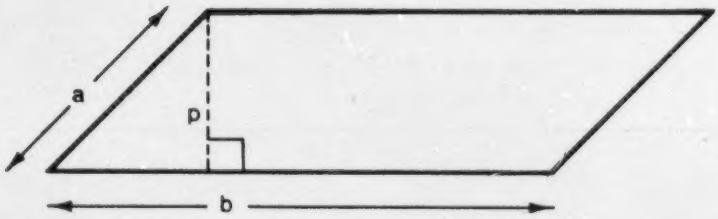
solution routine relevant to the goal is found in LTM, a problem is automatically recognized (Box F). Alternatively, a candidate solution may have been found at B, but the necessary conditions for running it may not be met (C), or the action may not be successful (E). In each case, a problem would be defined. This definition constitutes, in effect, a new goal, or a new encoding of the situation.

Wertheimer's descriptions of initial reactions to the parallelogram problem provide some examples of problem detection. In some of Wertheimer's experiments children who knew an algorithm for finding the area of a parallelogram presented in a horizontal display, as in Figure 2 at the top, were then given a vertically presented figure (as at the bottom of Figure 2). Some children immediately recognized the figure as one they had not "had yet" and refused to proceed. These children in effect failed to find a candidate solution routine. Others attempted to apply the standard routine. They dropped perpendiculars and then recognized an unfamiliar situation. These children apparently recalled the standard routine for finding area (at B of Figure 1), failed to note that it was inapplicable (C), tried it (D), and then found it unsuccessful (at E).

2. Feature scanning. Assume now that, by one route or the other, a problem has been detected; no immediately applicable or successful routine for the goal as initially represented has been found. It is characteristic of individuals who have detected problems to begin to scan the environment, apparently searching for clues. In functional fixedness problems, they typically attend to one after another of the objects available, apparently noting features of the objects. There is no evidence that there is an attempt to do an exhaustive scan, or to list all possible uses of the objects, as might seem to be suggested by theories of problem solving and creativity that stress fluency in producing many "unusual uses." Rather, this seems to be an idea-getting phase, a mapping of the environment, a highly heuristic and possibly partly random activity, much influenced by what first falls



Area = b x p

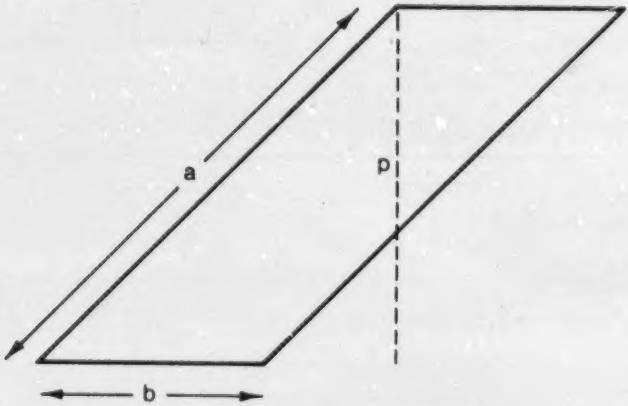


Figure 2. Area of a parallelogram problem.

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to hand or eye. Frequently coupled with this scanning of the physical environment is a questioning of the experimenter concerning the nature of the task requirements and restrictions on what can be done.

Boxes F through L of Figure 1 represent a sequence of events occurring during this feature scanning phase. The opening conditions (F) are set up by the results of the problem detection phase. Problem detection has resulted in a new definition of the goal. This condition initiates search of the task environment. The first action in this search is to select an object in the task environment (G). The term "object" is used to refer to both physical and symbolic objects, including verbal information from the experimenter. Feature detection activities can now begin (H). The noting of some feature in the external environment activates a new look at the contents of LTM (I). LTM is scanned for any item that "matches" or is linked to the feature noted. If an LTM item is found, the model suggests an evaluation of relevance to the goal as presently formulated (J). Essentially, the question asked is whether the item retrieved suggests (or constitutes) a solution. If an already organized solution is found, it is tested for applicability under present conditions (C) and, if possible, run (D) and tested for success (E). (Note that these actions return to a problem detection phase, thus signifying the constant interplay between problem detection and feature scanning activities in problem solving.) More typically, however, not a full, but a possible or partial solution is found (K), and this information is "kept in mind," that is, stored temporarily in working memory (L). At this point, as at several earlier points in the process, the individual may return to the beginning of the scanning activity.

Although it leaves the inner workings of many processes unspecified, this general model directs attention to important characteristics of the problem solving process. First, the process is extremely sensitive to the task environment. An initially empty WM is modified by a scanning of actually present objects or verbal instructions. What enters WM in this

00017

way may vitally affect the outcome of continuing problem solving efforts. Second, the process is characterized by a working back and forth between the current task environment and previously acquired knowledge (the "contents" of LTM). Feature detection leads to recall; recalled items are tested for relevance to the current situation. What actually enters WM is the result of this interaction. Finally, it is evident that the capacity of WM will vitally affect the problem solving process by limiting the amount of information noticed in the TE or accessed in LTM that can be kept accessible. Selective rehearsal strategies of some kind are thus likely to be crucial to successful problem solution.

3. Goal analysis. Note that the success or failure of the routines just described is dependent on finding an LTM item that matches the current definition of the problem. If the intial goal (at A) does not produce a match, it is only by creating a new goal (at F)--thus in effect, defining a new problem--that feature scanning activities can be initiated. Further, feature scanning alone does not ensure finding a routine. Successive redefinitions of the problem may be needed if that information or routines available in the individual's repertoire are to be recognized as relevant.

Much of the classical problem solving literature, particularly that drawn from the Gestalt tradition, focuses on this "restructuring" of the problem so that is becomes soluble. Emphasis in the Gestalt analyses is on the "insightful" nature of the process, the aha! nature of the experience, the way in which solution follows almost immediately upon recognition of a new form of the problem. Wertheimer's examples of this are familiar and dramatic. But Duncker is more explicit in suggesting the way in which analysis of the demands of a problem can lead to a solution. Examining the way in which one solution, considered and rejected, may lead to the next, Duncker (1945) speaks of the "process of solution as development of the problem" (p. 7). To quote him:

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> The final form of an individual solution is, in general, not reached by a single step from the original setting of the problem; on the contrary, the principle, the functional value of the solution, typically arises first, and the final form of the solution in question develops only as this principle becomes successively more and more concrete. In other words, the general or "essential" properties of a solution . . . precede the specific properties; the latter are developed out of the former. (pp. 7-8)

Duncker illustrates this process by presenting a "family tree" of solutions for the radiation problem (see Figure 3), each more specific than the one above it, but more general than those below. More modern terms for this redefinition process include identifying "differences" to be reduced, as in Newell and Simon's General Problem Solver (see Ernst & Newell, 1969) or "relating givens to unknowns," as in Greeno's (1973) discussion of problem solving. In Duncker's family tree it is possible to think of each solution possibility as a goal, directing search for a particular process. Each goal has subgoals that are explored until what Duncker calls a "block" is found (i.e., no productive ideas emerge). There is then a return to a higher level in the tree for a new start.

The important point concerning goal analysis is that goals are continually being redefined as a function of either memory search, usefulness of recalled routines, or noticed features of the environment. However a goal is generated, the contents of WM will eventually be modified. In addition, the task environment itself may be modified if actions performed result in a physical change in the presented stimuli. It is in both these senses that goal analysis can be thought to yield a "restructured" problem that permits use of already accessed routines or redirects the search for appropriate routines.

Some Studies on Invention and Assembly

In the course of this paper, we will briefly describe and discuss some of our own research on assembly processes in invention problems. The

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Tree of solutions to radiation problem. From "On Problem Solving" by Karl Duncker, Psychological Monographs, 1945, 58 (5, Whole No. 270). Copyright 1945 by the American Psychological Association. Removed to conform with copyright laws.

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tasks we work with are invention tasks of a particular kind. First, they are chosen so as to be relatively easily analyzed in terms of component routines that are subject to instruction. It is thus possible to assure, via instruction, that all subjects who enter the invention phase of an experiment are capable of calling upon and using these routines as separate processes, the assembly of which we can then observe.

Two tasks of this kind have been experimentally studied to date. One is a variant of Wertheimer's parallelogram problem. The second is derived from the task of multidigit addition involving carrying. The two problems share a common structure. In both cases the task as presented during the invention session has clear surface similarities to tasks encountered and successfully performed earlier in training. Thus, a "usual routine" for the class of problems exists in the subject's repertoire, and his or her first response is normally to attempt to apply this routine. However, in our problems, some aspect of the new invention task makes the usual routine inapplicable. The individual faced with the new task must, therefore, recognize that the usual routine is not applicable to the present case, thus detecting a problem, and then somehow construct a new routine by combining components in his or her repertoire. In each of the tasks the construction of the new routine is accomplished by applying a transformation routine that has the effect of changing the stimulus presented in the problem situation into one to which the usual routine does apply.

For the parallelogram problem, the usual routine is finding the area of rectangular figures by superimposing 1-inch cubes on the figures and then counting the cubes. Areas of two figures can be compared by putting cubes on both, counting both, and then comparing the numbers. This routine is simply an operationalizing, in a form suitable for young children, of the formula for area: Area = Length x Width. The routine is not applicable to nonrectangular figures because the blocks cannot be fit over them without hanging over the edges. The transformation routine that makes it

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possible to solve the problem is to cut the nonrectangle and rearrange the pieces into a rectangle. This must be done without adding or throwing away any pieces, thus maintaining equivalence between the presented and transformed stimuli.

The carrying problem involves the use of special materials that have been designed to represent the decimal and place value notation system for children just learning the number system. The materials (shown in Figure 4) consist of blocks: unit cubes, ten-bars, hundreds-squares. These blocks can be assigned to certain positions in a columnar array and thus display the value of the different columns in decimal notation. Any three-digit numeral can be represented in blocks on a three-column board. Conversely, any display of blocks that has nine or fewer blocks per column can be written as a numeral. Thus, in Figure 4, the display in row (a) stands for 275 and the display in row (b) stands for 409. Representing blocks with numerals 1 through 9 is taught to children as a "notation routine." If there are more blocks than nine in any column, however (as in row [c]), the notation routine would not be applicable, since only one digit is permitted in each column. To solve such a problem it is necessary to transform the stimuli. This can be accomplished by exchanging the blocks (ten ones for a ten-bar, for example) and placing the new block in its appropriate column. This is a concrete representation of the process we actually engage in when we carry in addition.

Figure 5 schematizes the task structure common to these two problems. A task stimulus and instructions are presented (A) and the subject tests the usual routine (B). Finding it inapplicable, the solution is to transform the task stimuli (D) while preserving important equivalences between the presented and transformed stimuli (E and F). Once the transformation has been made, it is now found possible to apply the usual routine (B). This is done (C), and the problem is solved. Note that the arrow between B and D in the figure is dotted. This represents the fact that the B - D connection

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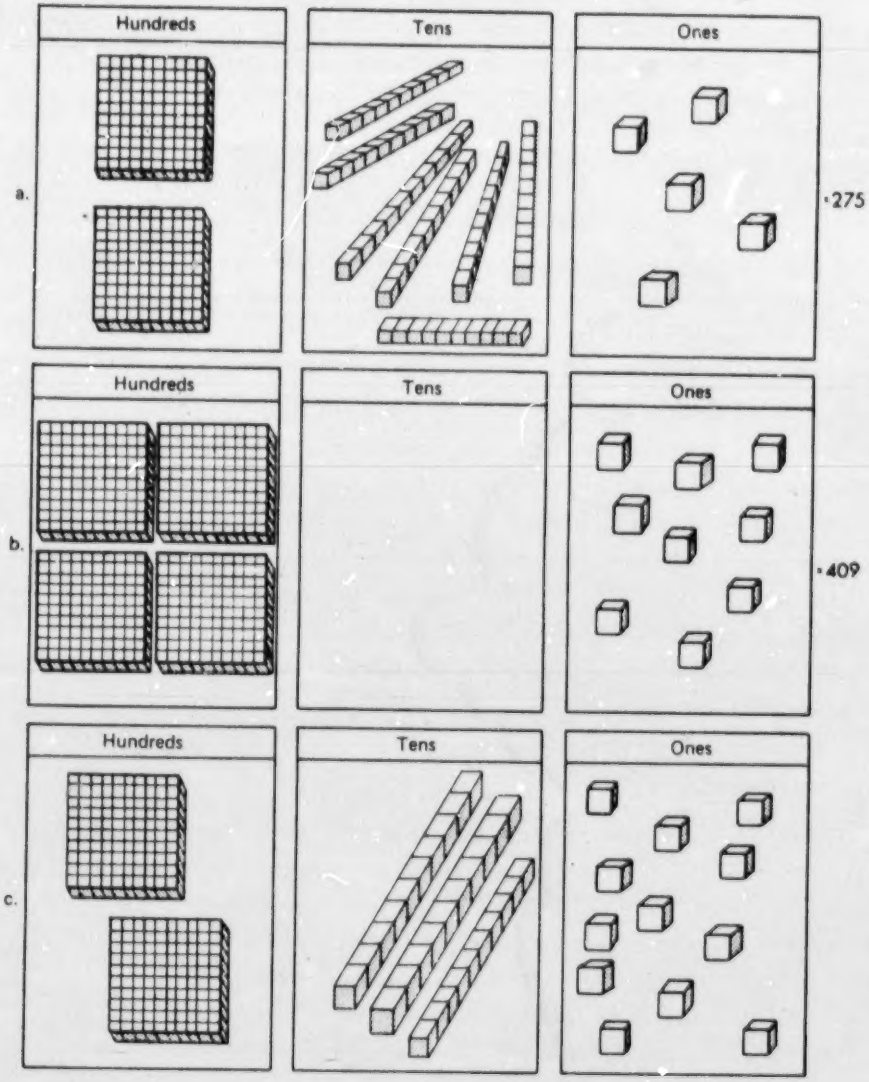

Figure 4. Block displays for notation problem.

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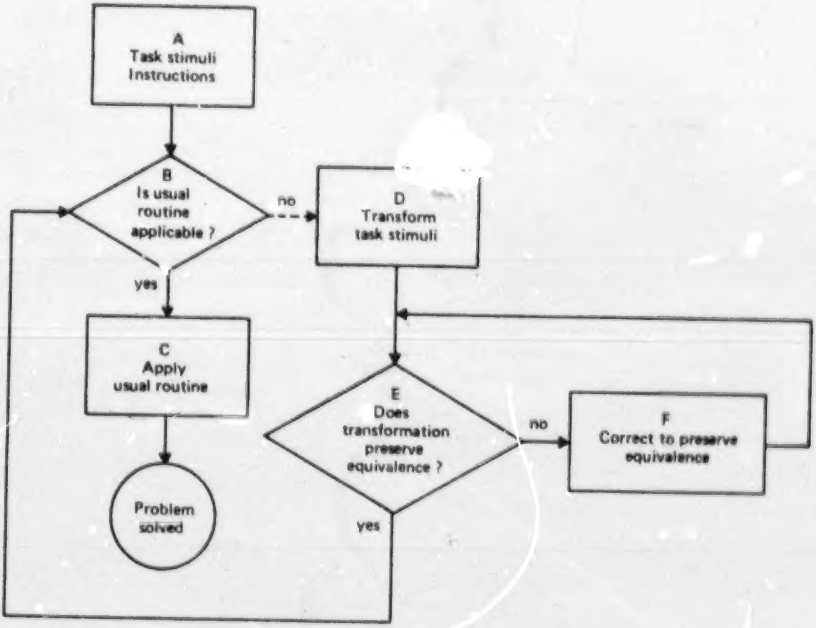

Figure 5. Schematic diagram of successful problem solving.

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is the invention that must be made by the individual. In our experiments, the links A - B and B - C are typically taught directly; so are the links D - E - F - E. But B - D is not taught, and the subject's solution of a problem consists of recognizing that when the usual routine is not applicable, the transformation routine can be applied. This invention consists of assembling two sets of routines, each well learned separately, but not previously used in combination.

Problem Detection

Our initial studies were concerned with problem detection. Most of the classical physical invention problems studied in the past contained clear environmental cues as to when the problem was solved. The strings were or were not tied together; the candle was stuck on the wall or not; there was or was not a place to hang one's hat. In our problems, by contrast, the criteria for an adequate problem solution are not as self-evident. Problem detection activity (indicated by the recognition of nonsolution) as a component of problem solving is thus more important.

An early exploratory study highlighted the effects of initial problem detection activity on later parts of the problem-solving process. The task was the notation (carrying) problem described earlier. First-grade children were divided into two training groups. In one group, the notation routine was taught by a series of games and practice exercises in which there were never more than nine blocks in any column. The children learned to write a three-digit numeral to represent the array without ever encountering the question of applicability of the routine. This was called the No-Detection group. The Detection group, by contrast, had notation routine training in which ten or more blocks occasionally appeared in either the tens or the ones column. When the child attempted to notate such a column, the experimenter stopped him, saying, "That column has more than nine blocks. You are only allowed to put one numeral in each column;

00025

so you do not have a way to do that column yet." Training on the exchange routine was identical for both groups.

Following training on the separate components, notation and exchange, there were ten invention trials. In these trials the children were presented arrays in which one column contained more than nine blocks and were asked to write the numeral that represented the display. Any child who did not spontaneously engage in exchange when encountering a notation problem with more than nine blocks in a column was prompted, using an increasingly explicit series of prompts. None of the children in the Detection group attempted to write two digits within a column--the typical "illegal" response made by untrained children. Thus, it was clear that Detection training quite clearly established a self-regulated problem detection routine that prevented the child's accepting a false solution. All children in the No-Detection group did attempt incorrect notation. These attempts were interrupted by the experimenter, who pointed out that only one digit per column was permitted and that there were too many blocks in the column.

To determine the effects of self-regulated problem detection, as opposed to external pointing out of the problem, we counted the number of trials on which a prompt to exchange was needed and also noted the specificity of the prompt that finally did produce exchange behavior. There was no clear difference between the groups in either measure. Thus, establishment of a strong problem detection routine may prevent acceptance of incorrect solutions, but it does not of itself make invention of new solutions any more or less likely. Instead, the strong effect of an established problem detection routine emerged in the way in which the exchange operations were carried out. Every child in the No-Detection group made what we called an "exchange error." An exchange error is essentially an incomplete exchange, one that does not preserve equivalence. Typically, ten unit-cubes are counted out and returned to the pool; but instead of picking up a ten-bar and adding it to the tens column, the child notates the

00026

ones column and goes on to the tens column as originally presented. No child in the Detection group made such an error. Thus, a well-established problem detection routine appears, on first look, to have its effect not so much in facilitating accessing of the transformation routine, but in carrying it out smoothly once accessed.

We have considered several possible explanations for this effect. Our currently favored hypothesis is that the problem detection routine leads to the establishment of a goal structure that calls on the exchange routine intact, while the lack of self-initiated problem detection sets up a goal structure that calls on pieces of the exchange routine, but not on the full routine itself. Figure 6 schematizes this hypothesis in terms of nested "stacks" of goals that activate and interrupt each other. There are two stacks, one for subjects in the Detection group, one for subjects in the No-Detection group. Movement through the stack is downward when new subgoals are being formulated, upward when subgoals are satisfied. Movement is always one goal down, or one up, at a time.

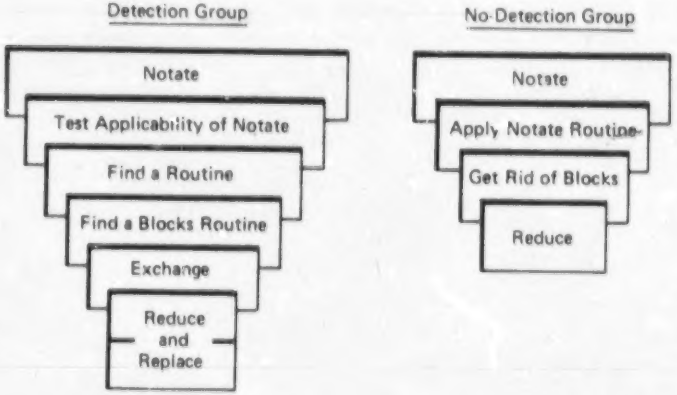

Figure 6. Hypothesized goal stacks for two invention groups.

00027

The two stacks share some goals (i.e., Notate and Reduce), but not all. Both groups begin with an active goal of Notate. For the Detection group, this activates a new goal, Test Applicability of Notate. When the Notate routine is found not to be applicable, new goals are formulated (the next two down in the list) that search for a routine that will solve the problem. This successive formulation of subgoals eventually produces the Exchange goal. Exchange will be satisfied only when both Reduce (a reduction of blocks in one column of the display board) and Replace (a corresponding replacement in the next column) have been satisfied. Thus, incomplete exchanges will not satisfy the Exchange goal. Only when Exchange is satisfied will the next goal up in the stack be reactivated. Successive goals will then be satisfied until Notate itself is again active.

The right side of Figure 6 shows the hypothesized goal stack for subjects who did not detect the problem, the No-Detection group in the experiment. Starting with the Notate goal, these subjects are hypothesized to immediately activate the goal Apply Notate Routine, which leads to an action of attempting to notate one of the columns. Our model assumes that, when stopped by the experimenter, these subjects encode the interruption as something like "Can't do it; too many blocks." Given this encoding, a reasonable subgoal to establish is Get Rid of Blocks, which interrupts the Apply Notate goal. This new goal produces a further subgoal, Reduce, a goal shared with the Detection group. The Reduce goal is satisfied once 10 blocks are eliminated from a column, but for the No-Detection subjects the interrupted goal for Reduce is not Exchange, but Get Rid of Blocks. Get Rid of Blocks is therefore reactivated, shown to be satisfied, and Apply Notate Routine is in turn reactivated. Replace--the second half of a complete exchange--is never generated as a goal by No-Detection subjects, because Exchange was never an active goal.

Our data suggest that problem detection is indeed an important part of the invention process in that the way problems are initially detected, and

00028

therefore encoded, will probably affect the quality of the observed solution behavior. Specifically, the present analysis proposes a relation between problem detection and the way the initial goal is analyzed. This suggests that any inclusive attempt to elucidate the nature of problem solving behavior must deal with questions regarding problem detection: To what extent is problem detection built into the external environment? To what extent are detection strategies common across problems, and to what extent are they task-specific? Are there stable individual differences in likelihood of detecting problems; if so, what processes underlie these differences?

Goals and the Analysis of the Problem

We next examined details of invention behavior when a problem-detection routine had been explicitly taught. For this purpose, the parallelogram problem was presented to fifth-grade students in a study conducted by Lynn Morris (Note 1). Morris began by writing a formal simulation analysis of the problem, using production system language. This analysis then guided development of a training procedure in which children were taught the conditions and actions that made up each task component.

The basic structure of the task, shown in Figure 7, is expressed in a production system (FIND AREA) that serves to organize and call on other production systems. These latter production systems define the separate "routines" that are taught to subjects. Three routines are called on by FIND AREA. These are TEST APPLICABILITY, USE BLOCKS, and TRANSFORM. Each was taught separately. The FIND AREA production rules were also taught, with the exception of FA4, the rule that calls on TRANSFORM when a nonrectangle ("no-figure") has been detected. FA4 corresponds to the B - D link in Figure 5. Construction and use of this rule constituted the invention that was sought.

00029

FA 1: ((GOAL: FIND.AREA) ⟶ LOOK AT.FIG)

If you want to find how big a figure is, look at the figure.

FA 2: ((GOAL: FIND.AREA) AND (FIG) ⟶ (GOAL: TEST. APPLICABILITY.OF.BLOCKS))

If you want to find how big a figure is, and you have a figure, then test to see if the blocks routine is applicable.

FA 3: ((GOAL: FIND.AREA) AND (YES.FIG) ⟶ (USE.BLOCKS (SATISFY GOAL))

If you want to find out how big a figure is and it is a figure to which the blocks routine is applicable (i.e., a 'yes-fig'), then use the blocks routine and the goal will be satisfied.

FA 4: ((GOAL FIND.AREA) AND (NO.FIG) ⟶ (GOAL: TRANSFORM))

If you want to find how big a figure is and it is a figure to which the blocks routine is not applicable (i.e., a 'no-fig'), then try to transform the figure.

Figure 7. Production system for FIND AREA.

Figure 8 summarizes the physical stimuli present and the experimenter's instructions for training each of the three routines and for the invention test. Twenty-four children were trained, each individually over a period of several weeks. On the invention test, three classes of subjects emerged. Five Inventors spontaneously and with relatively short latencies announced that the thing to do with the nonrectangle was cut it and then use the blocks. All other children began placing blocks on the nonrectangle. As the figure began to fill up with blocks, the experimenter intervened. She said, simply, "That's wrong, you can't do that." Two strikingly different responses to this feedback appeared. One group of seven children immediately cleared the figure of blocks. Another group (12 children) did not clear the blocks

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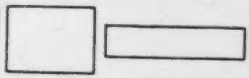
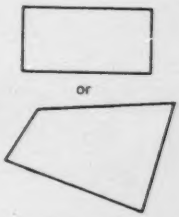
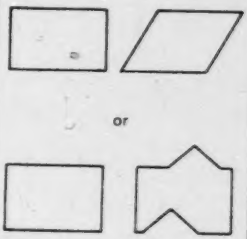
Routine	Physical Array	Experimenter's Instruction
USE BLOCKS		"Find which is bigger."
TEST FOR APPLICABILITY		"Does this figure have four right angles ?"
TRANSFORM	Various nonrectangular figures presented one at a time. Scissors handed to or pointed out to child. Booklet to paste figures into.	"To make this into a rectangle, you have to cut a part off and paste it on somewhere else. You must use all pieces and have four right angles."
INVENTION		"Find which is bigger."

Figure 8. Summary of stimulus conditions in the Morris experiment.

00031

but instead tried to rearrange them on the figure, apparently trying to make them "fit" better. Of the seven who cleared the figure, all subsequently thought of cutting the figure to produce a rectangle without any external prompting to do so. We call these children Assisted Inventors because, while they needed some feedback to distract them from the blocks routine, they in fact constructed and inserted FA4 for themselves. Equally striking, of the 12 who did not clear the figure, but instead rearranged the blocks, none ever mentioned cutting or tried to cut without direct prompting to do so by the experimenter. They failed to call on TRANSFORM on their own (i.e., they did not construct FA4), although they used the routine perfectly smoothly once prompted to do so. These children, half of our sample, must be termed Noninventors.

The Assisted Inventors and the Noninventors provide two clearly contrasting behavior patterns: When told "That's wrong" by the experimenter, those who were to invent cleared the figure; those who would not invent rearranged the blocks. What might have caused the different responses? And what differential effect did the responses have on the remainder of the solution process? With respect to the first question, a difference in currently active goals at the time of the experimenter's interruption might well account for the different responses. Given the same starting goal, Find Area, assume that some children take seriously the Test for Applicability of Blocks goal. When they place blocks on the nonrectangular figure, these children are really carrying out a concrete test, seeing if the blocks will fit. When the experimenter interrupts these children, they interpret her "Wrong" in terms of their current goal. Since they are testing to see if the blocks fit, they conclude that "wrong" means they do not fit, and therefore clear the figure. Assume now that some other children actually establish immediately a Use Blocks goal, and that this is what they are doing when they put the blocks in the nonrectangle. The experimenter's interruption is interpreted by these children in terms of the current goal,

00032

Use Blocks. Her "Wrong" is taken to mean that the blocks are placed incorrectly; thus the blocks are rearranged rather than removed.

With respect to the second question, the effect of clearing the blocks on the remainder of the solution process is to make visible again the non-rectangular figure. This allows the child to notice its features and, probably, to detect the "extra" piece at one end and the lack of it at the other, thus generating the idea of reconstructing the figure. Thus a new goal (Transform) would be formulated as a result of scanning the task environment. While this interpretation is not documented with clear-cut protocols or other data, it suggests further experiments in which periodic reconstruction of the physical display is a variable manipulated in order to determine the effects of a "return to an initial position."

The Task Environment:
Effects of External Cues on Accessibility of Routines

We have discussed the role of problem detection and goal analysis in promoting invention, suggesting at several points that these processes interact with feature detection and scanning of the task environment (TE) in the solution process. We now turn to a more direct consideration of the way in which the task environment features may affect problem solution.

Examination of Figure 8 reveals close similarity in several features between the cue conditions present for the training of the Use Blocks routine and for the invention test. In both cases there are two figures present; in both, the child is asked to "Find which is bigger." Thus, the invention task was presented in the context of TE cues very similar to the Use Blocks training task, and very different from the Transform training task, where only one figure was present and where the instructions were to cut it and make it into a rectangle. A recent study conducted by Tim Mulholland (1974) examined the effects of modifying the invention situation so that the cues were not so similar to those of block training. Half of 24

00033

fourth graders were given an invention task in which only a single figure (a parallelogram) was presented (thus more similar to transformation than to block training), and the instructions were simply "Find how big this is." The other half received the invention problem of the prior experiment where two figures, a rectangle and a parallelogram, were present. It was reasoned that the modified invention condition, with only a single figure that was not a rectangle, would not preemptively call the blocks routine. Rather, it would invite inspection of the situation and thus promote problem detection and a resultant goal analysis. Thus, conditions for accessing the transformation routine were thought to be stronger in the modified task. Mulholland's data suggest that the revised problem presentation does facilitate invention. Of the 12 children receiving the original two figures, only two invented, whereas six of the 12 children invented in the modified single-figure condition, and average time to solution was shorter for this latter group.

Optimizing Invention

We mention briefly one last study conducted by James Pellegrino and Margaret Schadler (Note 2). It represents an attempt to use everything we then knew or suspected concerning invention processes in designing a set of task conditions that would maximize the likelihood of invention. Pellegrino and Schadler's study again utilized the parallelogram task. Training on the Use Blocks, Test for Applicability, and Transform routines proceeded as in the prior experiments, but modifications were made in the invention phase. The most important change was designed to foster goal analysis by making children more self-conscious about the reasons for their actions. When the invention task was presented, the children were required to "look ahead" and to verbalize possible goals and strategies for meeting them before taking any action. Thus, in the invention situation the children were presented with two figures (either two parallelograms or a

00034

parallelogram and a rectangle) along with scissors and blocks, but the experimenter said only "What do you think I want you to do?" This forced the child to verbalize a goal for the problem. The experimenter elicited as many goal statements as the child was able to give. Only then was the next question asked: "Tell me how you would find which is bigger." The child was then required to state a plan of action and to tell how the planned action would help achieve the goal. Once the child had given one plan and a justification for that plan, he was allowed to proceed whether or not the experimenter thought it a good plan.

Under these "look ahead" conditions, 14 out of 16 children solved the problem, regardless of whether they were presented with two parallelograms or a parallelogram and a rectangle. Of the children who were not asked to verbalize goals and look ahead, but instead were asked simply to "Find which is bigger," only six out of 16 were able to solve the invention problem. In the latter group, there was slightly more invention on the part of those who were presented the two parallelograms, indicating that the task environment did have a facilitating effect in cueing the solution strategy, but the major conclusion drawn from the experiment (Pellegrino & Schadler, Note 2) was that "although the stimulus array [TE] is an important factor, the most powerful determinant of performance was the look-ahead verbalization activity, which effectively maximized solution for both arrays" (p. 19).

The results of this experiment suggest that the general strategy of planning ahead and considering alternative goals may be a very powerful component of problem solving. The looking ahead strategy appears to be both simple to use and easy to teach. It seems, in fact, that all that may be necessary is to remind people that they ought to consider their goals and possible actions; once reminded, they can access what they already have learned to do. Furthermore, it seems likely that the looking ahead strategy is generalizable across a variety of tasks, although this remains to be

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established experimentally. Such a strategy appears to be worth pursuing in instructional work designed to improve the ability to learn on one's own, and thus, by the definition adopted at the beginning of this chapter, to improve one's intelligence.

Implications for the Study of Intelligence

To recapitulate, we have argued that intelligence can be viewed as the ability to acquire new behavior in the absence of direct or complete instruction and that this ability involves processes that can facilitate the transition from simpler to more complex cognitive performance. We have used problem-solving behavior of the special kind we call invention as a window through which to examine the way in which individuals make transitions in competence on their own. Our strategy has been to present a model of this kind of problem solving which suggests the classes of processes that underlie the ability to learn without direct instruction, that is, to invent. Our obligation now is to suggest what this model implies for intelligence. We have argued that intelligence, defined as a transition process involving the assembly of components already in an individual's repertoire of competence, can be characterized in terms of three general kinds of activity: problem detection, feature scanning (noticing features in the environment) and analysis of goals. It is the way in which these activities are carried out that distinguishes between good and poor solution of invention problems. The potential of an individual as a problem solver is then a function of three things: (a) existing competence in task-specific subskills or components which need to be assembled in a solution; (b) general strategies with respect to problem detection, feature scanning, and goal analysis--i.e., the assembly processes themselves; and (c) the features of the particular task environment.

In our studies we have consistently assured the presence of the component subskills, thus allowing attention to be focused on assembly processes and task environment features. We have in some cases directly manipulated

00036

certain features of the task environment, and have shown (at least suggestively at this point) that task conditions that draw attention to certain features of the situation foster the finding of good solutions. One might say that "well-arranged" problem presentations lessen the demand on feature detection strategies and thus raise the probability of many individuals finding solutions. Somewhat more indirectly, we have manipulated the likelihood of problem detection behavior by teaching task-specific problem detection routines (i.e., the Tests for Applicability of the usual routines) that serve to focus the individual's attention on certain key features of the particular invention task environment to be encountered. These problem detection routines, too, have been shown to facilitate solution, probably by organizing subsequent search behavior in terms of an optimal goal structure that enables features of the task environment to be matched with available routines stored in memory.

It would appear, then, that if one wanted to help people perform as good problem solvers, one thing to do would be to put them into optimally designed environments--that is, environments that highlight relevant stimulus features and that directly suggest the locus of the problem. We might, if we became very intelligent about designing such environments, be able to create situations in which all of our subjects seemed to be highly intelligent. But such a feat of engineering would miss the point of our concern. As we said at the outset of this paper, intelligence is precisely the ability to acquire new abilities under less than optimal environmental conditions, conditions where the appropriate solution routines are not directly prompted or specifically taught. Under such conditions, which are quite general in the normal course of life, the burden of detecting relevant features, analyzing problems, and establishing appropriate goals rests with the individual. This implies that to account for intelligence, we have to address problem detection, feature scanning strategies, and goal analysis strategies as generalized competencies of the individual and as competencies in which individuals differ.

00037

The heart of our effort lies in our attempt to develop a model for a pervasive kind of human behavior. In so doing, we purposely have used concepts drawn from current information processing theories in the hope that many of the unanalyzed components of our model will be opened for inspection by others, and thus perhaps give more explicitness to the preliminary notions presented here. In our own experimental work a key tactic is to use instruction in the hypothesized processes as a means of verifying the reality of those processes. The aim of such research is not to investigate the question of instructability as such; rather, the instructability of particular hypothesized processes is assumed, and these processes are taught. If the instruction (which may need to be of an extended and varied nature) is successful, and if the instructed individual behaves in ways similar to individuals who have become good problem solvers on their own, then presumptive evidence will exist in favor of the reality of the processes we have hypothesized.

In the studies reported in this paper the instructional efforts were for the most part limited to routines specific to the particular tasks involved. A job now ahead is to devise means of instructing people in the processes we have hypothesized as general to problem solution and to evaluate the effects of such instruction across a variety of task environments. We have made a pilot attempt in this direction, with respect to the goal analysis component of our model--namely, the requirement in the Pellegrino-Schadler study that children engage in goal analysis by verbalizing plans of action and predicting expected outcomes. The effect across tasks of such self-consciousness about goals and probable outcomes remains to be examined. Similar efforts with respect to teaching generalized strategies of problem detection and heuristics of feature scanning are also required next steps. To the extent that such a research program based on instruction proves tractable, we hope at a later date to be able to point with more certainty to one critical set of the processes that constitute intelligence and perhaps to give increased operational meaning to the possibilities for increasing intelligence via instruction.

00038

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